Wednesday, February 14, 2024 11:23 AM

SPECIAL CASE $F: A \subseteq \mathbb{R}^n \to \mathbb{R}$ CONT

WITH $A = \mathbb{R}^n$.

THM SPECIALIZES TO

COROLLARY $F : \mathbb{R}^n \to \mathbb{R}$. THE FOLLOWING ARE EQUIVALENT:

1) F IS CONT ON $\mathbb{R}$

2) $\forall B \subseteq \mathbb{R}$, B OPEN $\Rightarrow F^{-1}[B] = \mathbb{R}^n \cap B_1 = B_1$ OPEN

B_1 OPEN

3) $\forall C \subseteq \mathbb{R}$, CLOSED $\Rightarrow$

$\Rightarrow F^{-1}[C] = \mathbb{R}^n \cap C_1 = C_1$ CLOSED.

C_1 CLOSED

EX.

$A = \{(x,y) \in \mathbb{R}^2 ; \; 17x^3y^{11} + 103x^5y^7 + 90x^{17}y^{25} < 17\}$

IS A OPEN? YES

WHY? CONSIDER

$F : \mathbb{R}^2 \to \mathbb{R}$ S.T.

$F(x,y) = 17x^3y^{11} + 103x^5y^7 + 90x^{17}y^{25}$

IS POLYNOMIAL FUNCTION $\Rightarrow F$ CONTINUOU FUNCT!!!

$A = \{(x,y) \in \mathbb{R}^2 ; \; F(x,y) < 17\}$

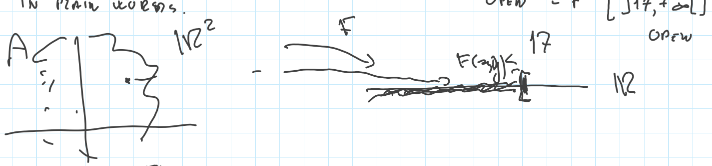
> 17

$A = \{(x,y) \in \mathbb{R}^2 ; F(x,y) > 17\}$

OPEN $= F^{-1}[\,]17, +\infty[\,]$

OPEN

IN PLAIN WORDS.



THAT IS

$A = \{(x,y) \in \mathbb{R}^2 ; \; F(x,y) < 17\} =$

$= F^{-1}[\,]-\infty, 17[\,]$ IS OPEN !!!!

OPEN SET IN $\mathbb{R}$

2) SIMILARLY

$B = \{(x,y) \in \mathbb{R}^2 ; \; 17x^5y^6 + 115x^{13}y^{25} + 9x^3y^{27} \leq 1\}$

AGAIN LET $F: \mathbb{R}^2 \to \mathbb{R}$ S.T.

$F(x,y) = 17x^5y^6 + 115x^{13}y^{25} - 8x^3y^{27}$

POLYNOMIAL $\to$ CONT.

$B = \{(x,y)\in\mathbb{R}^2;\ F(x,y) \leq 1\} =$ $\geq 1 \to B = \{(x,y);\ F(x,y)\geq 1\}$ CLOSED!!... $= F^{-1}[[1,+\infty[]$ CLOSED

$= F^{-1}\left[\,]-\infty, 1]\,\right]$ IS CLOSED

$]-\infty,1]$ CLOSED IN $\mathbb{R}$

(3) $A = \{(x,y)\in\mathbb{R}^2;\ x^3y^{11} + 127x^7y^{95} - 11x^5y^{12} = 12\}$

AGAIN $F(x,y): \mathbb{R}^2 \to \mathbb{R}$

WHERE $F(x,y) = x^3y^{11} + 127x^7y^{95} - 11x^5y^{12}$

F IS POLYNOMIAL $\Rightarrow$ F CONTINUOUS

$A = \{(x,y)\in\mathbb{R}^2;\ F(x,y) = 12\} =$

$= F^{-1}[\{12\}]$ CLOSED

(F CONT, $\{12\}$ CLOSED)

EX / APPL

$A = \{(x,y)\in\mathbb{R}^2;\ x^2+y^2 \neq \frac{1}{n^2},\ n\in\mathbb{Z}^+\}$

IS A MEASURABLE. IS A BORELIAN $\mathbb{R}^n$

NOTICE THAT

$A = B^c$

WHERE

$\to B = \{(x,y)\in\mathbb{R}^2;\ x^2+y^2 = \frac{1}{n^2},\ n\in\mathbb{Z}^+\}$

$A = \mathbb{R}^2 - B$

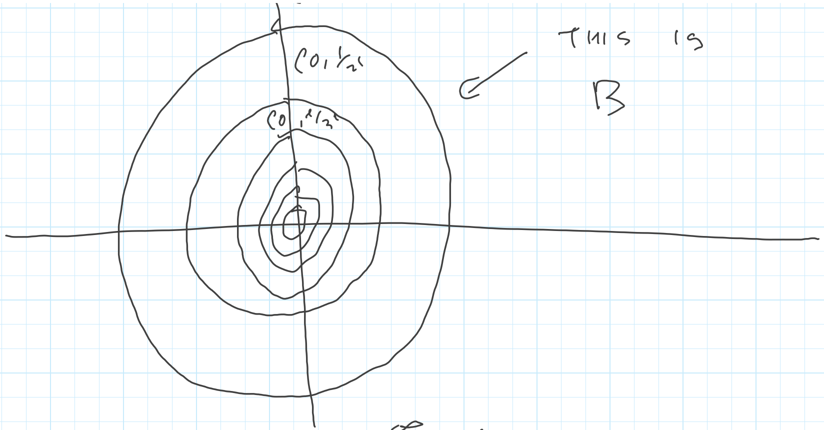
$B = \{(x,y)\in\mathbb{R}^2;\ x^2+y^2 = \frac{1}{n^2},\ n\in\mathbb{Z}^+\}$

COUNT

$= \bigcup_{n\in\mathbb{Z}^+} \{(x,y)\in\mathbb{R}^2;\ x^2+y^2 = \frac{1}{n^2}\}$

$\mathbb{R}^2$

(0,1)



THIS IS B

SO $B = \bigcup_{n=1}^{\infty} \underline{\{(x,y)\in\mathbb{R}^2 ; x^2+y^2 = 1/n^2\}}$ BOREL

CLOSED

BOREL

IN PASSING, IS B CLOSED? NO

WHY? SINCE (0,0) IS AN ACC. POINT FOR B BUT $(0,0) \notin B$.

SINCE $A = B^c = \mathbb{R}^n - B$ A BOREL !!!

BOREL BOREL

STOP

QUESTIONS???

BYEBYE, GOOD WEEKEND