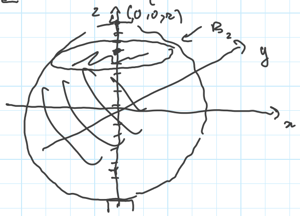
Wednesday, February 14, 2024 11:39 AM

EX 3 $B = \{(x,y,z) \in \mathbb{R}^3 ; x^2+y^2+z^2 \le r^2\} \subseteq \mathbb{R}^3$



B CLOSED $\Rightarrow$ B MEASURABLE

$r=2, s=1$

$\mathbb{R}^3 = \mathbb{R}^2 \times \mathbb{R}^1$

$(x,y) \quad z$

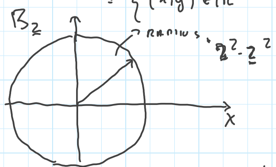
$\mu_3(B)$

GIVEN $z \in \mathbb{R}$, WHAT IS

$B_z = \{(x,y) \in \mathbb{R}^2 ; (x,y,z) \in B\} =$

$= \{(x,y) \in \mathbb{R}^2 ; x^2+y^2+z^2 \le r^2\}$

$= \{(x,y) \in \mathbb{R}^2 ; x^2+y^2 \le r^2 - z^2\}$



$\mu_2(B_z) = \pi(r^2 - z^2)$

THEN $S_B = \{z \in \mathbb{R} ; -r < z < r\}$

$\mu_3(B) \overset{\text{Thm}}{=} \int_{S_B} \mu_2(B_z)\, dz =$

$= \int_{S_B =]-r,r[} [\pi(r^2 - z^2)]\, dz$

$= \int_{-r}^{r} (\pi(r^2 - z^2))\, dz =$

$= \left[\pi r^2 z\right]_{z=-r}^{z=r} - \left[\frac{1}{3}\pi z^3\right]_{z=-r}^{z=r} =$

$= 2\pi r^3 - \frac{2}{3}\pi r^3 = \boxed{4/3\, \pi r^3}$

THE GENERAL TONELLI / FUBINI THM FOR INTEGRAL

THM LET $A \subseteq \mathbb{R}^n$ MEASURABLE.

1) LET $f: A \subseteq \mathbb{R}^n \to \mathbb{R}$ MEAS + NN FUNCT (TONELLI VERSION)

AGAIN $\mathbb{R}^n = \mathbb{R}^2 \times \mathbb{R}^3$ 2+3=n

i) $x \in \mathbb{R}^2$ $A_x = \{ y \in \mathbb{R}^3 ; (x,y) \in A \} \subseteq \mathbb{R}^2$

SECTION

ii) $S_A = \{ x \in \mathbb{R}^2 ; \mu_3^*(A_x) > 0 \}$ SUPPORT

$S_A^0 = \{ x \in \mathbb{R}^2 ;$ A_x NOT MEAS. BAD SUPPORT

WE ALREADY KNOW S_A, S_A^0 MEAS + $\mu_2(S_A^0) = 0$

NOW FOR EVERY $x \in S_A \setminus S_A^0$

$f_x(y) = f(x,y)$ $y \in A_x \subseteq \mathbb{R}^3$

$f_1 : x \in S_A \setminus S_A^0 \subseteq \mathbb{R}^2 \longrightarrow f_1(x) \overset{DEF}{=} \int_{A_x} f(x,y)\,dy$ IS FUNCT OF x

WELL

$$\int_{A \subseteq \mathbb{R}^n} f = \int_{S_A \setminus S_A^0} f_1(x)\,dx = \int_{S_A \setminus S_A^0 \subseteq \mathbb{R}^2} \left(\int_{A_x} f(x,y)\,dy \right) dx$$

IF WE DELETE HP f NN ??

FROM THE TONELLI ???

$\Downarrow$

FUBINI FORM

$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$ MEAS !!

DELETE : NN

IF WE ALREADY KNOW

$\int_A f < +\infty$

$\Downarrow$

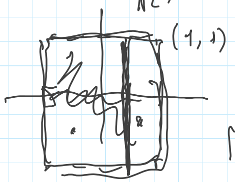
THE SAME STATEMENT AS TONELLI.

EX 1 $A = \{ (x,y) \in \mathbb{R}^2 ; |x| < 1, |y| < 1 \}$

$\mathbb{R}^3$ $f : A \to \mathbb{R}$, $f(x,y) = x+y$ IS NOT NONNEGATIVE

CONTINUOUS

$\bar{A}$ COMPACT



$|x| < 1$ $A_x = \{ y \in \mathbb{R} ; |y| \leq 1 \}$ $\Downarrow$

$\int_A f < +\infty$ FUBINI APPLIES

$S_A =]-1, 1[$

$$\int_A f = \int_A (x+y)\,dx\,dy = \int \left(\int_{A_x =]-1,1[} f(x,y)\,dy \right) dx =$$

$S_A =]-1,1[$

$$\int_A f = \int^1 \left(\int^1 (x+y)\,dy \right) dx =$$

$$= \int_{-1}^{1} \left(\left[xy + \frac{y^2}{2} \right]_{y=-1}^{y=1} \right) dx =$$

$$= \int_{-1}^{1} 2x \, dx = \left[x^2 \right]_{x=-1}^{x=1} = \underline{0} \; !!!!$$

THE COURSE IS OVER

QUESTIONS

BYE BYE