Wednesday, March 27, 2024 11:07 AM

SIA $\underline{n} = \{1, 2, \dots, n\}$

$$\mathbb{P}(\underline{n}) = \{A ; A \subseteq \underline{n}\}$$

CALCOLARE $|\mathbb{P}(\underline{n})| = ?$

1) $A \subseteq \underline{n}$, $\chi_A : \underline{n} \to \{0,1\}$

FUNZ. CARATTERISTICA

OVE $\chi_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$ $\quad x \in \underline{n}$

2) $\chi : \underline{n} \to \{0,1\}$

$$\mathrm{supp}(\chi) = \{x \in \underline{n} ; \chi(x) = 1\}$$

ORA

$$\to A \subseteq \underline{n} \xrightarrow{1)} \chi_A \xrightarrow{2)} \mathrm{supp}(\chi_A) = A$$

$$\chi \xrightarrow{2)} \mathrm{supp}(\chi) \xrightarrow{1)} \chi_{\mathrm{supp}(\chi)} = \chi$$

1) & 2) SONO <u>INVERSE</u> $\Rightarrow$ 1) & 2) SONO <u>BIEZIONI</u>

$$\mathbb{P}(\underline{n}) \underset{\text{su}}{\overset{1-1}{\longleftrightarrow}} \{\chi : \underline{n} \to \{0,1\}\}$$

$$\# \mathbb{P}(\underline{n}) = \# \{\chi ; \underline{n} \to \{0,1\}\} = 2^n$$

<u>COEFF. BINOMIALI</u>

$n, k \in \mathbb{N}$

$\binom{n}{k} \overset{\text{DEF}}{=}$ # k - SOTTOINSIEMI DI UN n - INSIEME

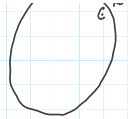
<u>PROBLEMA</u> $\binom{n}{k} = ?$

<u>SOL RICORSIVA</u> $\binom{n}{0} = 1 \qquad \binom{0}{k} = \delta_{0k} = \begin{cases} 1 & 0 = k \\ 0 & 0 \neq k \end{cases}$

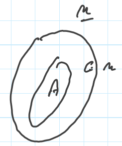
DATI $n, k \in \mathbb{N}$

QUANTO VALE $\binom{n}{k}$???

$\underline{n}$

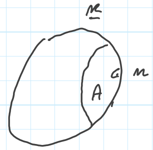


SIA $A \subseteq \underline{n}$, $|A| = k$ RISP. AL [illegible] ELEMENT



1) $n \notin A$

SONO $\binom{n-1}{k}$



2) $n \in A$

SONO $\binom{n-1}{k-1}$

SI HA $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

SOL RICORSIVA

$$\begin{cases} \binom{n}{0} = 1 \quad \binom{0}{k} = \delta_{0k} & \text{COND. INIZIALI} \\ \binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} & \underline{\text{RICORRENZA}} \end{cases}$$

MATRICI BIINFINITE

$M : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{R}$, $M : (n,k) \longrightarrow M(n,k) \in \mathbb{R}$

	0	1	2	..	k	.....
0						
1						
2						
⋮						
n					M(n,k)	
..						

BASO DEI BINOMIALI

$M : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{R}$, $M(n,k) \overset{DEF}{=} \binom{n}{k}$ CHI E'?

	0	1	2		k-1	k	....
0	1	0	0		0	0	....
1	1					⋮	
2	1					⋮	
⋮						⋮	
n-1	1				$\binom{n-1}{k-1}$ +	$\binom{n-1}{k}$	
n	1	.	.		..	$\binom{n}{k}$	
⋮	⋮						

$\binom{n}{0} = 1$

$\binom{0}{k} = \delta_{0k}$

$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

	0	1	2	3	4	5	...	
0	1	0	0	0	0	0		
1	1	1	0	0	0	0		
2	1	2	1	0	0	...		
3	1	3	3	1	0	...		
4	1	4	6	4	1	0	...	
5	1	5	10	10	5	1	0	...