Wednesday, March 27, 2024 11:07 AM

COEFF. BINOMIALI

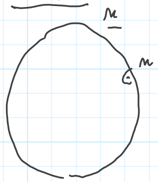
$n, k \in \mathbb{N}$

$$\binom{n}{k} \overset{DEF}{=} \# \ k\text{-sottoinsiemi di un } n\text{-insieme.}$$

CALCOLO ?? — **RICORSIONE !!**

$$\begin{cases} \binom{n}{0} = 1 \qquad \binom{0}{k} = \delta_{0k} \overset{DEF}{=} \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \\ \binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \end{cases}$$

BAD ELEMENT.

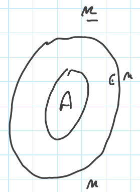


BAD ELEMENT. FISSO $A \subseteq \underline{n}$, $|A| = k$.

HO DUE CASI DISGIUNTI ED ESAUSTIVI

1) $n \notin A$

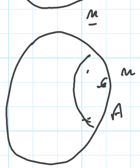
$\# = \binom{n-1}{k}$



$$\Rightarrow \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

2) $n \in A$

$\# = \binom{n-1}{k-1}$



PROBLEMA NUOVO

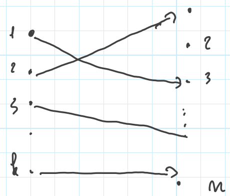
$$\binom{n}{k} \overset{?}{=} \text{FORMA CHIUSA} \ ???$$

I k-SOTTOINSIEMI SONO LE "PECORE" ???

CHI SONO LE "ZAMPE" ???

$F : \underline{k} \xrightarrow{1-1} \underline{n}$

F 1



$\Downarrow$

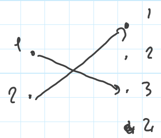
$Im\, F \subseteq \underline{n}$

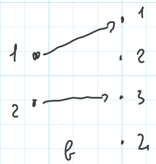
$|Im\, F| = k$

È CHIARO CHE DUE DIVERSE $F, G: \underline{k} \xrightarrow{1-1} \underline{n}$

POSSONO ESSERE TALI CHE $Im\, F = Im\, G$.

ES $k = 2$ $n = 4$





$F \rightsquigarrow Im\, F = \{1,3\} = Im\, G = \{1,3\}$ G

QUANTE SONO LE FUNZIONI $F: \underline{k} \xrightarrow{1-1} \underline{n}$

t.c. $Im\, F = A \subseteq \underline{n}$ FISSATO TALE CHE $|A| = k$?

SONO $k!$

2 AMRE

$$\binom{n}{k} = \frac{\#\{F: \underline{k} \xrightarrow{1-1} \underline{n}\}}{k!} = \frac{(n)_k}{k!} \qquad \text{OK!!!}$$

SI NOTI

$$\boxed{\frac{(n)_k}{k!}} = \frac{n(n-1)\cdots(n-k+1)\cdot(n-k)!}{k!\,(n-k)!} = \frac{n!}{k!(n-k)!}$$