Wednesday, March 27, 2024 11:09 AM

Begin at 9.10

SERIE FORMALI (DI POTENZE)

SIA

$(a_n)_{n \in \mathbb{N}} = (a_0, a_1, \ldots, a_n, \ldots)$ una successione

con $a_n \in \mathbb{R}$.

Sia t una variabile "formale".

ASSOCIAMO

$$\alpha(t) = \sum_{n=0}^{\infty} a_n t^n$$

"serie formale associata alla successione."

SIA $\mathbb{R}[[t]]$ L'INSIEME DELLE SERIE FORMALI

+) $\alpha(t) = \sum_{n=0}^{\infty} a_n t^n$, $\beta(t) = \sum_{n=0}^{\infty} b_n t^n$

ALLORA $\alpha(t) + \beta(t) \overset{\text{DEF}}{=} \sum_{n=0}^{\infty} (a_n + b_n) \cdot t^n$

·) $\lambda \in \mathbb{R}$

$$\lambda \alpha(t) = \sum_{n=0}^{\infty} (\lambda a_n) \cdot t^n$$

$\mathbb{R}[[t]]$ È UNO SPAZIO VETTORIALE

×) PRODOTTO

DEFINIAMO

$$\alpha(t) \times \beta(t) \overset{\text{DEF}}{=} \gamma(t) = \sum_{n=0}^{\infty} c_n t^n$$

OVE $c_n = \sum_{k=0}^{n} a_k b_{n-k}$. !!!

$(\mathbb{R}[[t]], +, \cdot, \times)$ UNA ALGEBRA

MATRICI BIINFINITE E SERIE GENERATRICI PER RIGHE.

$M : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{R}$, $M : (n,k) \longrightarrow M(n,k) \in \mathbb{R}$.

PER $n \in \mathbb{N}$. LA SERIE GENERATRICE (RIGA) DI M

È

$$M_n(t) \overset{\text{DEF}}{=} \sum_{k=0}^{\infty} M(n,k)\, t^k$$

AD ES $M : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{R}$, $M(n,k) = \binom{n}{k}$

	0	1	2	3	..	k	0	
0	1	0	0	0		0	..	$\longrightarrow M_0(t) = 1$

0	1	0	0	0	0	
1	1	1	0	-	-	$\to M_1(t) = 1+t$
2	1	2	1	0	-	$\to M_2(t) = 1+2t+t^2$
3	1	3	3	1 $\cdots$ 0		$\to M_3(t) = 1+3t+3t^2+t^3$
$\vdots$						
n						

MATRICE RICORSIVA

$M : \mathbb{N}\times\mathbb{N} \to \mathbb{R}$ RICORSIVA $\overset{DEF}{\iff}$

$$M_n(t) = M_1(t)\,M_{n-1}(t) \iff M_n(t) = (M_1(t))^n$$

THM SIA $M(n,k) = \binom{n}{k}$.

ALLORA È RICORSIVA.

PROOF.

ABBIAMO $M_1(t)\cdot M_{n-1}(t) \overset{DEF}{=}$

$$(1+t)\left(\sum_{k=0}^{\infty}\binom{n-1}{k}t^k\right) \overset{DEF}{=}$$

$$= \gamma(t) = \sum_{k=0}^{\infty} c_k\, t^k \quad \text{OVE}$$

$$c_n = \sum_{h=0}^{k} M(1,h)\,M(n-1,k-h) \quad \text{PER DEF}$$

$$= 1\cdot M(n-1,k) + 1\cdot M(n-1,k-1)$$

$$= 1\cdot\binom{n-1}{k} + 1\cdot\binom{n-1}{k-1} \overset{THM}{=} \binom{n}{k}$$

DA CUI $\gamma(t) = \sum_{k=0}^{\infty} c_k t^k = \sum_{k=0}^{\infty}\binom{n}{k}t^k \overset{DEF}{=} M_n(t)$.

NE SEGUE IL THM BINOMIALE:

$$(1+t)^n \overset{thm}{=} \sum_{k=0}^{n}\binom{n}{k}t^k.$$

CONVOLUZIONE DI VANDERMONDE (BINOMIALE)

$M(n,k) = \binom{n}{k}$ CASO BINOMIALE

$$M_1(t) = 1+t, \quad M_n(t) \overset{DEF}{=} \sum_{k=0}^{\infty}\binom{n}{k}t^k \overset{THM}{=} (1+t)^n = (M_1(t))^n$$

ORA $i,j \in \mathbb{Z}^+$ t.c. $i+j=n$.

MA ORA

$$M_n(t) = (M_1(t))^{n} = (M_1(t))^{i}\,(M_1(t))^{j} =$$

$$= M_i(t)\, M_j(t)$$

ORA

$$M_i(t)\, M_j(t) \overset{DEF}{=} g(t) = \sum_{k=0}^{\infty} e_k\, t^k ,$$

OVE

$$e_k = \sum_{h=0}^{k} M(i,h)\, M(j,k-h) \qquad \underline{DEF}\ (\dagger)$$

MA,

$$g(t) = M_i(t)\, M_j(t) = (M_1(t))^{i}\,(M_1(t))^{j} =$$

$$= M_n(t) \overset{DEF}{=} \sum_{k=0}^{\infty} \binom{n}{k} t^k$$

CIOÈ $\quad e_k = \binom{n}{k}$

MA $(\dagger)$

$$e_k = \sum_{h=0}^{n} M(i,h)\, M(j,k-h) =$$

$$= \sum_{h=0}^{n} \binom{i}{h} \binom{j}{k-h}$$

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CIOÈ VANDERMONDE CLASSICO

$$\sum_{h=0}^{n} \binom{i}{h}\binom{j}{k-h} = \binom{n}{k}$$