Wednesday, March 27, 2024 11:10 AM

RICORDO, $n, k \in \mathbb{N}$

$$\left\langle\!\left\langle {n \atop k} \right\rangle\!\right\rangle \overset{\text{DEF}}{=} \#\ k\text{-multisets su un } n \text{ insieme.}$$

THM

$$\left\langle\!\left\langle {n \atop k} \right\rangle\!\right\rangle \overset{\text{THM}}{=} \frac{\langle n \rangle_k}{k!} = \frac{\overbrace{(n+k-1)(n+k-2)\cdots(n+1)n}\ (n-1)!}{k!\ \ (n-1)!}$$

$$= \frac{(n+k-1)!}{k!\ (n-1)!} = \binom{n+k-1}{k} = \binom{n+k-1}{n-1}$$

FORMA RICORSIVA PER $\left\langle\!\left\langle {n \atop k} \right\rangle\!\right\rangle$.

$$\left\langle\!\left\langle {n \atop 0} \right\rangle\!\right\rangle = 1 \quad \forall n, \qquad \left\langle\!\left\langle {0 \atop k} \right\rangle\!\right\rangle = \delta_{0k} = \begin{cases} 1 & 0 = k \\ 0 & \text{ALTRIMENTI} \end{cases}$$

$$M : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{R}, \qquad M(n,k) \overset{\text{DEF}}{=} \left\langle\!\left\langle {n \atop k} \right\rangle\!\right\rangle.$$

	0	1	2	3	4	- -	k - - - -
0	1	0	0	0	0		0 - - -
1	1	1	1	1	1		1
2	1	2	3	4	5	- -	
3	1	3	6	10	. . .	- -	
4	1						
$n-1$	1	- - -					
n	1	- - -					$\left\langle\!\left\langle {n \atop k} \right\rangle\!\right\rangle$

BAD ELEMENT BATTEZZO $n \in \underline{n} = \{1, \dots, n\}$

HO I SEGUENTI CASI DISGIUNTI ED ESAUSTIVI

$f(n) = k \longrightarrow \left\langle\!\left\langle {n-1 \atop 0} \right\rangle\!\right\rangle$

$+$

$f(n) = k-1 \longrightarrow \left\langle\!\left\langle {n-1 \atop 1} \right\rangle\!\right\rangle$

$+$

$f(n) = k-2 \longrightarrow \left\langle\!\left\langle {n-1 \atop 2} \right\rangle\!\right\rangle$

$+$

- - -

$+$

$f(n) = 0 \longrightarrow \left\langle\!\left\langle {n-1 \atop 0} \right\rangle\!\right\rangle$

$$=$$

$$\left\langle {n \atop k} \right\rangle$$

<u>THM</u>

$$\left\langle {n \atop 0} \right\rangle = 1 \qquad \left\langle {0 \atop k} \right\rangle = \delta_{0k}$$

$$\left\langle {n \atop k} \right\rangle = \sum_{i=0}^{k} \left\langle {n-1 \atop i} \right\rangle = \left\langle {n-1 \atop k} \right\rangle + \left\langle {n \atop k-1} \right\rangle$$

$$\begin{array}{c|cccccc}
 & 0 & 1 & 2 & -- & k-1 & k \\
\hline
n-1 & \times & \times & \times & & \times & \left\langle {n-1 \atop k} \right\rangle \\
n & & & & & \left\langle {n \atop k-1} \right\rangle & \left\langle {n \atop k} \right\rangle
\end{array}$$

	0	1	2	3	4				
0	1	0	0	0	0				
1	1	1	1	1	1	1	1	1	...
2	1	2	3	4	5				
3	1	3	6	10	15	-	-	—	
4	1								
5	1								

SIA $M : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$, $M(n,k) = \left\langle {n \atop k} \right\rangle$

<u>LE SERIE GENERATRICI PER RIGHE :</u>

$$M_n(t) \overset{\text{DEF}}{=} \sum_{k=0}^{\infty} \left\langle {n \atop k} \right\rangle t^k$$

SI NOTI CHE

$$M_1(t) = 1 + t + t^2 + \dots + t^k \dots - = \sum_{k=0}^{\infty} t^k$$

MA $\displaystyle\sum_{k=0}^{\infty} t^k = \frac{1}{(1-t)}$? <u>PERCHE'</u> ???

$$(1+t+t^2+\dots+t^k+\dots)(1-t) = 1 \qquad ???$$

$$1 - t + t - t^2 + t^2 + \dots = 1 \qquad \text{YES}$$

ORA

THM $M : \mathbb{N}\times\mathbb{N} \to \mathbb{R}$, $M(n,k) \doteq \left\langle {n \atop k} \right\rangle$

È UNA MATRICE RICORSIVA

CON SERIE DI RICORSIONE $M_1(t) = \sum_{k=0}^{\infty} t^k = \frac{1}{(1-t)}$,

CIOÈ

$$M_n(t) \stackrel{DEF}{=} \sum_{k=0}^{\infty} \left\langle {n \atop k} \right\rangle t^k \stackrel{THM}{=} (M_1(t))^n = \frac{1}{(1-t)^n}$$

DA DIMOSTRARE!!