Wednesday, March 27, 2024 11:11 AM

DA DIMOSTRARE:

$$M_1(t)\,M_{n-1}(t) \overset{THM}{=} M_n(t) \overset{DEF}{=} \sum_{k=0}^{\infty} \left\langle {n \atop k} \right\rangle t^k$$

$$\left(1+t+t^2+\dots+t^k\dots\right)\left(\sum_{k=0}^{\infty}\left\langle {n-1 \atop k}\right\rangle t^k\right) =$$

$$= g(t) = \sum_{k=0}^{\infty} c_k\, t^k \quad \text{ove}$$

$$c_k \overset{DEF}{=} \sum_{h=0}^{k} \left\langle {1 \atop h}\right\rangle \left\langle {n-1 \atop k-h}\right\rangle = \sum_{h=0}^{k} \left\langle {n-1 \atop k-h}\right\rangle =$$

$$= \left\langle {n-1 \atop 0}\right\rangle + \left\langle {n-1 \atop 1}\right\rangle + \dots + \left\langle {n-1 \atop k}\right\rangle \overset{\text{RIC 1)}}{\underset{THM}{=}} \left\langle {n \atop k}\right\rangle$$

PERCIÒ

$$M_1(t)\,M_{n-1}(t) = \sum_{k=0}^{\infty}\left\langle {n \atop k}\right\rangle t^k \overset{DEF}{=} M_n(t) \qquad \text{QED !!!}$$

OK

CONSEGUENZA (VANDERMONDE PER $\left\langle {n \atop k}\right\rangle$)

$$M(n,k) \overset{DEF}{=} \left\langle {n \atop k}\right\rangle .$$

$i, j \in \mathbb{Z}^+$

$i + j = n$

$$M_n(t) \overset{THM}{=} \left(M_1(t)\right)^n =$$

$$= \left(M_1(t)\right)^i \left(M_1(t)\right)^j =$$

$$= M_i(t)\, M_j(t)$$

$$M_i(t)\,M_j(t) = g(t) = \sum_{k=0}^{\infty} c_k\, t^k$$

OVE

$$c_k = \boxed{\sum_{h=0}^{k}\left\langle {i \atop h}\right\rangle \left\langle {j \atop k-h}\right\rangle = \left\langle {n \atop k}\right\rangle}$$

CIOÈ: $\quad i + j = n$

THM

$$\sum_{h=0}^{k}\left\langle {i \atop h}\right\rangle \left\langle {j \atop k-h}\right\rangle = \left\langle {n \atop k}\right\rangle . \quad \boxed{\text{Ⓑ}}$$

CONVOLUZIONI TRA BINOMIALI "SEGNATI"

& MULTIINSIEMISTICI

SIA $\quad \alpha^*(t) \overset{DEF}{=} (1-t) \quad \Rightarrow$

$$\Rightarrow (a'(t))^n = (1-t)^n = \sum_{k=0}^{n} (-1)^k \binom{n}{k} t^k \; !!!$$

TRE CASI

1) $n > m$

$$\frac{(1-t)^n}{(1-t)^m} = (1-t)^n \cdot \frac{1}{(1-t)^m}$$

$$= \left(\sum_{k=0}^{\infty} (-1)^k \binom{n}{k} t^k\right)\left(\sum_{k=0}^{\infty} \left\langle {m \atop k} \right\rangle t^k\right) =$$

$$= y(t) = \sum_{k=0}^{\infty} e_k \, t^k$$

OVE

$$e_k = \sum_{h=0}^{k} (-1)^h \binom{n}{h} \left\langle {m \atop k-h} \right\rangle \qquad (\dagger)$$

MA

$$y(t) = (1-t)^n \cdot \frac{1}{(1-t)^m} = (1-t)^{n-m} \overset{n>m}{=}$$

$$= \sum_{k=0}^{\infty} \underbrace{(-1)^k \binom{n-m}{k}}_{\text{"}e_k = (\dagger)} t^k$$

PERCIO'

$$n > m \Rightarrow$$

$$\sum_{h=0}^{k} (-1)^h \binom{n}{h} \left\langle {m \atop k-h} \right\rangle = (-1)^k \binom{n-m}{k}$$

2) $n = m$

$$(1-t)^n \cdot \left(\frac{1}{(1-t)^n}\right) = 1$$

$$\left(\sum_{k=0}^{\infty} (-1)^k \binom{n}{k} t^k\right)\left(\sum_{k=0}^{\infty} \left\langle {n \atop k} \right\rangle\right) =$$

$$= y(t) = \sum_{k=0}^{\infty} e_k \, t^k$$

OVE

$$e_k = \sum_{h=0}^{k} (-1)^h \binom{n}{h} \left\langle {n \atop k-h} \right\rangle = \delta_{k0} \begin{cases} 1 & k=0 \\ 0 & \text{ALTRIMENTI} \end{cases}$$

3) $m > n$

$$(1-t)^n \cdot \frac{1}{(1-t)^m} = \frac{1}{(1-t)^{m-n}} = \sum_{k=0}^{\infty} \left\langle {m-n \atop k} \right\rangle t^k$$

$$g(t) = \sum_{k=0}^{\infty} e_k \, t^k$$

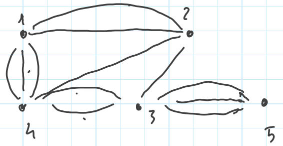
ove

$$e_k = \sum_{h=0}^{k} (-1)^h \binom{n}{h} \left\langle {m \atop k-h} \right\rangle \overset{\text{TH}}{=} \left\langle {m-n \atop k} \right\rangle$$

cioè $\underline{m > n} \Rightarrow$

$$\underline{\sum_{h=0}^{k} (-1)^h \binom{n}{h} \left\langle {m \atop k-h} \right\rangle = \left\langle {m-n \atop k} \right\rangle}$$

MULTIGRAFO



$$\left\langle {\binom{5}{2} \atop 12} \right\rangle$$

DEFINIZIONE FORMALE

$$G_\rho = (V, E_\rho)$$

ove

G_ρ: multigrafo

V: insieme dei vertici

E_ρ multiset t.c.

OVE

$$E_\rho : \{A \subseteq V ; |A| = 2\} \longrightarrow \mathbb{N}$$

PROBL 1 Quanti sono i multigrafi su n vertici $(V = \{1, 2, \dots, n\})$

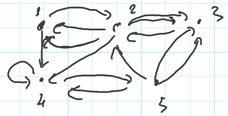
Sono # multiset su $\underline{n}$ =

$$\left\langle {n \atop 0} \right\rangle + \left\langle {n \atop 1} \right\rangle + \left\langle {n \atop 2} \right\rangle + \dots + \left\langle {n \atop k} \right\rangle + \dots = +\infty$$

PROBL 2 Quanti sono i multigrafi su n vertici con k lati?

$$\left\langle {\binom{n}{2} \atop k} \right\rangle$$

MULTIDIGRAFI



DEF FORMALE

$$\vec{G}_\rho = (V, \vec{E}_\rho)$$

INS VERTICI

OVE $\vec{E}_\rho : V \times V \longrightarrow \mathbb{N}$ MULTISET

QUANTI SONO I MULTIDIGRAFI CON k FRECCIE ???

$$\left\langle \begin{matrix} n^2 \\ k \end{matrix} \right\rangle \quad !!!$$