PRINCIPIO DI INVERSIONE DI MÖBIUS INSIEMISTICO DUALE

THM In $(\mathbb{P}(S), \supseteq)$ si ha:

Se $f, g : \mathbb{P}(S) \to \mathbb{R}$ tali che

$$\sum_{A \supseteq B} f(A) = g(B) \quad \forall B \in \mathbb{P}(S)$$

Allora

$$f(B) = \sum_{A \supseteq B} (-1)^{|A|-|B|} g(A) \quad \forall B \in \mathbb{P}(S).$$

PROBL DI DERANGEMENT GENERALIZZATI

k, n

$$d_{n,k} \overset{\text{DEF}}{=} \# \left\{ \sigma : \underline{n} \xrightarrow[1-1]{\text{SU}} \underline{n} \; ; \; \sigma \text{ HA ESATTAMENTE } k \text{ PTI FISSI} \right\}$$

RICORDIAMO: $d_{n,k} = \binom{n}{k} d_{n-k}$

ove $d_0 = 1$, $d_1 = 0$ e $d_n = (n-1)(d_{n-2} + d_{n-1})$

SOL RICORSIVA

FORMA CHIUSA

Ricordo, $\sigma : \underline{n} \xrightarrow[1-1]{\text{SU}} \underline{n}$ sia

$$FIX(\sigma) = \{ i \in \underline{n} \; ; \; \sigma(i) = i \}.$$

Fissato $B \subseteq \underline{n}$

$$(*) \quad \left\{ \sigma : \underline{n} \xrightarrow[1-1]{\text{SU}} \underline{n} \; ; \; FIX(\sigma) \supseteq B \right\} = \dot{\bigcup_{A \supseteq B}} \left\{ \sigma : \underline{n} \xrightarrow[1-1]{\text{SU}} \underline{n} \; ; \; FIX(\sigma) = A \right\}$$

Passando alle cardinalità: siano

$$g(B) = \# \left\{ \sigma : \underline{n} \xrightarrow[1-1]{\text{SU}} \underline{n} \; ; \; FIX(\sigma) \supseteq B \right\}$$

$$f(A) = \# \left\{ \sigma : \underline{n} \xrightarrow[1-1]{\text{SU}} \underline{n} \; ; \; FIX(\sigma) = A \right\}$$

$(*)$ DIVENTA:

$$g(B) = \sum_{A \supseteq B} f(A) \quad \forall B \subseteq \underline{n}$$

$\Downarrow$ INV. MOBIUS DUALE

$$f(B) = \sum_{A \supseteq B} (-1)^{|A|-|B|} g(A) \qquad (**)$$

SIA B FISSATO, $B \subseteq \underline{n}$, $|B| = k$

QUANTO VALE $f(B) = ?$

DA $(**)$ SI HA

$$f(B) = \sum_{\substack{A \supseteq B \\ A \subseteq \underline{n}}} (-1)^{|A|-k} g(A)$$

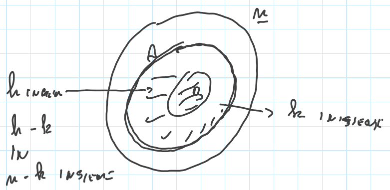
$$= \sum_{h=k}^{n} \Big(\sum_{\substack{A \supseteq B \\ |A| = h}} (-1)^{h-k} g(A) \Big) \qquad (**)$$

$(n-h)!$

???

QUANTI SONO GLI $A \subseteq \underline{n}$, $A \supseteq B$, $|B| = k$, $|A| = h$

QUINDI SONO $\binom{n-k}{h-k}$!!



$(**)$ DIVENTA:

$$\sum_{h=k}^{n} (-1)^{h-k} \binom{n-k}{h-k} (n-h)!$$

$$= \sum_{h=k}^{n} (-1)^{h-k} \frac{(n-k)!}{(h-k)!\cancel{(n-h)!}} \cancel{(n-h)!}$$

PER $B \subseteq \underline{n}$, $|B| = k$

$$f(B) = \sum_{h=k}^{n} (-1)^{h-k} \frac{(n-k)!}{(h-k)!}$$

MOLTIPLICANDO PER # SCELTE DI B

$$d_{k,n} = \binom{n}{k} \cdot \sum_{h=k}^{n} (-1)^{h-k} \frac{(n-k)!}{(h-k)!} =$$

$$= \frac{n!}{\quad} \sum^{n} (-1)^{h-k} \frac{\cancel{(n-k)!}}{\quad}$$

THM

$$\frac{}{k!(n-k)!} \sum_{h=k} \qquad \frac{}{(h-k)!}$$

$$= \frac{n!}{k!} \sum_{h=k}^{n} \frac{(-1)^{h-k}}{(h-k)!} \qquad (\times \times)$$

ORA $d_n = d_{0,n} \Rightarrow k=0$ IN $(\times\times)$

SI HA $d_n = n! \sum_{h=0}^{n} \frac{(-1)^h}{h!}$

P_n (σ NON HA PT FISSI) =

$$= \frac{d_n}{n!} = \sum_{h=0}^{n} \frac{(-1)^h}{h!}$$

CHI È L'ASINTOTICA, CIOÈ

$$P_\infty = \lim_{n \to \infty} P_n = \lim_{n \to \infty} \left(\sum_{h=0}^{n} \frac{(-1)^h}{h!} \right)$$

$$= \sum_{h=0}^{\infty} \frac{(-1)^h}{h!}$$

LA TAYLOR DI e^x È

$$e^x = \sum_{h=0}^{\infty} \frac{x^h}{h!} \Rightarrow$$

PER $x = -1$

$$\sum_{h=0}^{\infty} \frac{(-1)^h}{h!} = e^{-1} = \frac{1}{e}$$