Wednesday, March 27, 2024 11:21 AM

SIEVE METHODS

TRE PROTAGONISTI

1) Ω INSIEME (SAMPLE SPACE)

2) $A_1, A_2, \dots, A_m$ $\quad (A_i \subseteq \Omega \quad i = 1, 2, \dots, m)$

"EVENTI PROIBITI"

3) $\underline{m} = \{1, 2, \dots, m\}$

PROBL $\quad \left| \Omega \setminus \bigcup_{i=1}^{m} A_i \right| = ???$

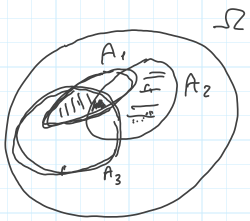
PRODOTTI COMPLETI

DATO $T \subseteq \underline{m}$ $\quad$ PROD. COMPL. ASSOCIATO A $T \subseteq \underline{m}$

$$\boxed{\bigcap_{i \in T} A_i \cap \bigcap_{i \notin T} A_i^c} =$$

$$= \{ x \in \Omega \ ; \ x \in A_i, \forall i \in T \ \& \ x \notin A_i \ \forall i \notin T \}$$

EX $\quad m = 3$



SIA:

$T = \{1, 3\}$

$A_1 \cap A_3 \cap A_2^c$

$T = \{2\}$

$A_2 \cap A_1^c \cap A_3^c$

$T = \{1, 2, 3\}$

$A_1 \cap A_2 \cap A_3$

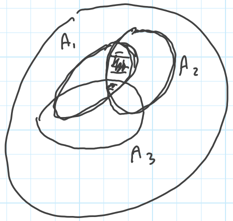
ORA

PROP. $\quad S \subseteq \underline{m}$

$$\bigcap_{i \in S} A_i \overset{\text{THM}}{=} \dot{\bigcup_{\substack{T \subseteq \underline{m} \\ T \supseteq S}}} \left(\bigcap_{i \in T} A_i \cap \bigcap_{i \notin T} A_i^c \right) \qquad (+)$$

EX $\quad m = 3$ $\qquad S = \{1, 2\}$

Ω



$$\bigcap_{i\in S} A_i = A_1 \cap A_2$$
$$= (A_1 \cap A_2 \cap A_3^c)\ \dot\cup$$
$$\dot\cup\ (A_1 \cap A_2 \cap A_3)$$

OVE $A_1 \cap A_2 \cap A_3^c$ IL PROD. COMPL. PER $T=\{1,2\}$

& $A_1 \cap A_2 \cap A_3$ IL PROD. COMPL. PER $T=\{1,2,3\}$

CONSIDERIAMO DUE FUNZIONI

$$f, g : \mathbb{P}(\underline{n}) \to \mathbb{R}$$

COSÌ DEFINITI:

$\forall T \subseteq \underline{n}$

$$f(T) = \left| \bigcap_{i\in T} A_i \cap \bigcap_{i\notin T} A_i^c \right|$$

&

$\forall S \subseteq \underline{n}$

$$g(S) = \left| \bigcap_{i\in S} A_i \right|$$

QUINDI, PASSANDO ALLE CARDINALITÀ, LA $(\dagger)$

$$\sum_{\substack{T\subseteq \underline{n}\\ T\supseteq S}} f(T) = g(S) \qquad \forall S \subseteq \underline{n}.$$

$\Downarrow$ INV. MÖBIUS DUALE

$$f(S) = \sum_{\substack{T\subseteq \underline{n}\\ T\supseteq S}} (-1)^{|T|-|S|} g(T)$$

$$\parallel$$

$$\left| \bigcap_{i\in S} A_i \cap \bigcap_{i\notin S} A_i^c \right| = \sum_{k=m}^{n} (-1)^{k-m} \left(\sum_{\substack{T\subseteq \underline{n}\\ |T|=k}} \left| \bigcap_{i\in T} A_i \right| \right)$$ SE $|S|=m$

PERCIÒ, DATO $S \subseteq \underline{n}$, $|S|=m$

$$(2)\quad \left| \bigcap_{i\in S} A_i \cap \bigcap_{i\notin S} A_i^c \right| = \sum_{k=m}^{n} (-1)^{k-m} \left(\sum_{\substack{T\subseteq \underline{n}\\ |T|=k}} \left| \bigcap_{i\in T} A_i \right| \right)$$

ORA, SIA $S=\emptyset$

$$\bigcap_{i\in n} A_i^c = \Omega \setminus \bigcup_{i=1}^{n} A_i$$

IN $(*)$ CON $S=\emptyset$

$$\left|\Omega \setminus \bigcup_{i=1}^{n} A_i\right| \overset{(*)\,\text{THM}}{=} \sum_{k=0}^{n} (-1)^k \underbrace{\left(\sum_{\substack{T\subseteq n \\ |T|=k}} \left|\bigcap_{i\in T} A_i\right| \right.}$$

<u>THM DI SYLVESTER</u>

$$= \sum_{k=0}^{n} \underline{(-1)^k} \; S_k \leftarrow \text{NUMERO DI SYLVESTER}$$

<u>ES</u> $n=3$ $\quad \Omega, \quad A_1, A_2, A_3$

$$\left|\Omega \setminus \bigcup_{i=1}^{3} A_i\right| = \left|\Omega \setminus (A_1 \cup A_2 \cup A_3)\right| \overset{\text{THM}}{=}$$

$$= \underbrace{|\Omega|}_{k=0} - \underbrace{|A_1| - |A_2| - |A_3|}_{k=1} +$$

$$+ \underbrace{|A_1\cap A_2| + |A_1\cap A_3| + |A_2\cap A_3|}_{k=2} - \underbrace{|A_1\cap A_2 \cap A_3|}_{k=3} \; .$$ <u>C.V.D.</u>