FORMULA DI SYLVESTER:

$$\left|\Omega \setminus \bigcup_{i=1}^{n} A_i\right| \overset{\text{THM}}{=} \sum_{k=0}^{n} (-1)^k \left(\sum_{\substack{T\subseteq \underline{n}\\ |T|=k}} \left|\bigcap_{i\in T} A_i\right|\right)$$

X X

FORMULA DI CH. JORDAN

Ω INS. $A_1, A_2, \ldots, A_n$ EVENTI PROIBITI

$\underline{n} = \{1,2,\ldots,n\}$

PER $m = 0,1,\ldots,n$ DEFINIAMO

$$e_m = \quad \text{(VALENZA)}$$

$\overset{\text{DEF}}{=}$ # degli elementi di Ω CHE APPARTENGONO AD ESATTAMENTE m EVENTI PROIBITI.

CHIARAMENTE

$$e_0 = \left|\Omega \setminus \bigcup_{i=1}^{n} A_i\right|$$

CALCOLIAMO

$$e_m = \sum_{\substack{S\subseteq \underline{n}\\ |S|=m}} f(S) =$$

$$= \sum_{\substack{S\subseteq \underline{n}\\ |S|=m}} \left(\sum_{T\supseteq S} (-1)^{|T|-|S|} g(T)\right)$$

$$= \sum_{k=m}^{n} (-1)^{k-m} \binom{k}{m} \sum_{\substack{T\supseteq S\\ |T|=k}} g(T)$$

$$= \sum_{k=m}^{n} (-1)^{k-m} \binom{k}{m} \underbrace{\sum_{|T|=k} \left|\bigcap_{i\in T} A_i\right|}_{\text{"}S_k}$$

$$= \sum_{k=m}^{n} (-1)^{k-m} \binom{k}{m} S_k$$

IN CONCL. THM (CH. JORDAN)

$$e_m = \sum_{k=m}^{n} (-1)^{k-m} \binom{k}{m} S_k$$!!!!

CHE QUINDI, PER $m = 0$

$$\left|\Omega \setminus \bigcup_{i=1}^{n} A_i\right| = e_0 = \sum_{k=0}^{n} (-1)^k S_k \qquad \text{(SYLVESTER)}$$

DERANGEMENT GENERALIZZATI

$$\Omega = \{ \sigma : \underline{n} \xrightarrow[1-1]{su} \underline{n} \,;\ \sigma \text{ permutazione} \}$$

FISSATO k, VOGLIAMO CALCOLARE

QUANTE SONO LE n-PERMUTAZIONI CON

ESATTAMENTE k PTI FISSI, CIOÈ $d_{n,k}$!!!

PER OGNI $i = 1, 2, \ldots, n$

$$A_i \subseteq \Omega = \{ \sigma : \underline{n} \xrightarrow[1-1]{su} \underline{n} \}$$

OVE $A_i := \{ \sigma \in \Omega \,;\ \sigma(i) = i \}$

$$d_{n,k} = e_k \overset{\text{CH JORDAN}}{=} \sum_{h=k}^{n} (-1)^{h-k} \binom{h}{k} \underbrace{\binom{n}{h} (n-h)!}_{S_k}$$

$$= \sum_{h=k}^{n} (-1)^{h-k} \frac{\cancel{h!}}{(h-k)!\,k!} \frac{n!}{\cancel{h!}\,\cancel{(n-h)!}} \cancel{(n-h)!}$$

$$= \frac{n!}{k!} \sum_{h=k}^{n} \frac{(-1)^{h-k}}{(h-k)!}$$

FORMULA GIÀ VISTA PER INVERSIONE