Wednesday, March 27, 2024 11:22 AM

LA FUNZIONE Φ DI EULERO

DEF

$$\Phi : \mathbb{Z}^+ \longrightarrow \mathbb{Z}^+$$

TALE CHE, $\forall n \in \mathbb{Z}^+$

$$\Phi(n) \overset{DEF}{=} \# \{ m \in \underline{n} \ ; \ MCD(n,m) = 1 \}$$

AD ES, $\Phi(10) = ?$

$$1, \cancel{2}, 3, \cancel{4}, \cancel{5}, \cancel{6}, 7, \cancel{8}, 9, \cancel{10}$$

QUINDI $\Phi(10) = 4$.

TEOREMA DI EULERO SIA $n \in \mathbb{Z}^+$ E SIA NOTO

$$n \doteq p_1^{i_1} p_2^{i_2} \cdots p_r^{i_r} \qquad p_1, \dots, p_r \text{ PRIMI}$$

TEOR. FOND. ARITMETICA

ALLORA

$$\Phi(n) = n \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_r}\right) \quad !!!$$

PROOF DA SYLVESTER

PONGO $\Omega = \underline{n} = \{1, 2, \dots, n\}$

E CONSIDERO r EVENTI

$$A_i \overset{DEF}{=} \{ m \in \underline{n} \ ; \ p_i \mid m \} \qquad i = 1, 2, \dots, r$$

QUINDI

$$\Phi(n) = \left| \Omega \setminus \bigcup_{i=1}^{r} A_i \right| \overset{SYLVESTER}{=}$$

$$= \sum_{k=0}^{r} (-1)^k \sum_{\substack{T \subseteq \underline{r} \\ |T| = k}} \left| \bigcap_{i \in T} A_i \right|$$

ADESSO, SIA $T = \{ j_1, j_2, \dots, j_k \}$

$\left| \bigcap_{j \in T} A_j \right|$ SONO I NUMERI $m \in \underline{n}$ DIVISIBILI PER $p_{j_1}, p_{j_2}, \dots, p_{j_k}$

DA CUI

$$\left|\bigcap_{j\in T} A_j\right| = \frac{n}{p_{j_1} p_{j_2} \cdots p_{j_k}}.$$

ES $n = 30$, $n = 2 \cdot 3 \cdot 5$ $p_1 = 2$, $p_2 = 3$, $p_3 = 5$

$T \in \{1,2,3\}$

$T = \{1,3\}$

$$\left|\bigcap_{j\in T} A_j\right| = A_1 \cap A_3 \Rightarrow |A_1 \cap A_3| = \frac{30}{10}$$

$\boxed{???}$

VEDI DOPO

QUINDI

$$\phi(n) \overset{\text{SYLVESTER}}{=} n\Big(\underset{k=0}{1} \overbrace{- \frac{1}{p_1} - \frac{1}{p_2} - \cdots - \frac{1}{p_r}}^{k=1} + \overset{k=2}{\sum} \frac{1}{p_i p_j} - \overset{k=3}{\sum} \frac{1}{p_i p_j p_k} + \cdots$$

$$= n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_r}\right)$$ CVD.

PERCHÉ (???)

$n = 30$ $n = 2 \cdot 3 \cdot 5$ $p_1 = 2$, $p_2 = 3$, $p_3 = 5$

$A_1 \cap A_3$ SONO I NUMERI FINO A 30 DIVISIBILI PER 2 E PER 5 CIOÈ DIVISIBILI PER 10

$$|A_1 \cap A_3| = \frac{30}{2 \cdot 5} = \frac{30}{10} = 3$$

$10 \cdot 1 = 10 \leq 30$, $10 \cdot 2 = 20 \leq 30$, $10 \cdot 3 = 30 \leq 30$

AD ES

$n = 30 = 2 \cdot 3 \cdot 5$

$p_1 = 2$, $p_2 = 3$, $p_3 = 5$ $r = 3$

$$\phi(30) = 30\Big(\underset{k=0}{1} \overbrace{- \frac{1}{2} - \frac{1}{3} - \frac{1}{5}}^{k=1} + \overset{k=2}{\left(\frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 5} + \frac{1}{3 \cdot 5}\right)} - \overset{k=3}{\frac{1}{2 \cdot 3 \cdot 5}}\Big)$$

$= 30\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)$

CVD

ESEMPIO